Concentrated Review of Trouble Topics for Exam 1

Squeeze Theorem

1. Use the squeeze theorem to compute $\lim_{h\to 0}\frac{Cos(h)-1}{h}$. [Hint: Consult the appropriate handout]
2. Use the squeeze theorem to compute $\lim_{h\to 0}\frac{Sin(h)}{h}$. [Hint: Consult the appropriate handout]
3. Calculate $\lim_{x\to 0} xCos\left(e^{Sin(1/x)}\right)$.
4. Define $f(x)=\begin{cases} 1+x^4 & if \quad x \quad is \quad irrational \\ 1+2x^4 & if \quad x \quad is \quad rational \end{cases}$. Find $\lim_{x\to 0} f(x)$.
5. Use the squeeze theorem to compute $\lim_{x\to\infty}\frac{x^3 Sin(1/x)+1}{x^6+10}$.
6. Suppose $-1\le f(x)\le x^2-2x$ for all x. Compute $\lim_{x\to 1} f(x)$.

Intermediate Value Theorem

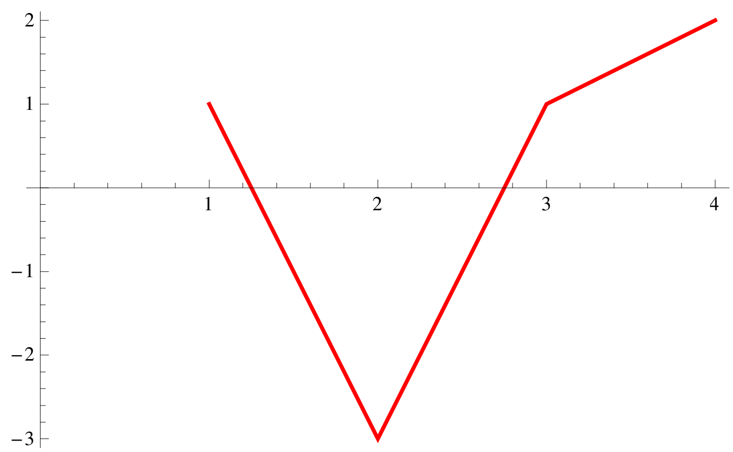
1. Let *f* : [0, 1] → [0, 1] be a continuous function. The formula for this function is not given, but we know that $0\le f(x)\le 1$. Must there be a solution to the *equation* $f(x)=x^3$? Why or why not? Justify your answer. [**Please note that I am NOT claiming that** *f* (x) **is the function** x^3. For example, $f(x)=xCos\left(\frac{1}{1+x^2}\right)$ satisfies the conditions of this problem. In that case the equation $f(x)=x^3$ becomes $xCos\left(\frac{1}{1+x^2}\right)=x^3$]
2. True or false, the function $Cos(x)$ is cubing at least one number?
3. True or false, the equation $Cos\left(x^2\right)=x^7$ has at least one solution?
4. Let *f*, g : [0, 1] → [0, 1] be two continuous functions, such that $0\le f(x)\le 1$ and $0\le g(x)\le 1$. Must the equation $f(x)=g(x)$ have a solution? If not, what additional conditions must we impose on *f* and *g* to make sure the equation does have solutions?
5. Let *f* : (0, 1] → [0, 1] be a continuous function. The formula for this function is not given, but we know that $0\le f(x)\le 1$ and that *f* (0) is undefined. Must there be a solution to the *equation* $f(x)=x$? Why or why not? Justify your answer.

6. Let $f : [0, 1) \to [0, 1]$ be a continuous function. The formula for this function is not given, but we know that $0 \le f(x) \le 1$ and that $f(1)$ is undefined. Must there be a solution to the *equation* $f(x) = x$? Why or why not? Justify your answer.
7. Let $f : (0, 1) \to [0, 1]$ be a continuous function. The formula for this function is not given, but we know that $0 \le f(x) \le 1$ and that $f(0), f(1)$ are undefined. Must there be a solution to the *equation* $f(x) = x$? Why or why not? Justify your answer.

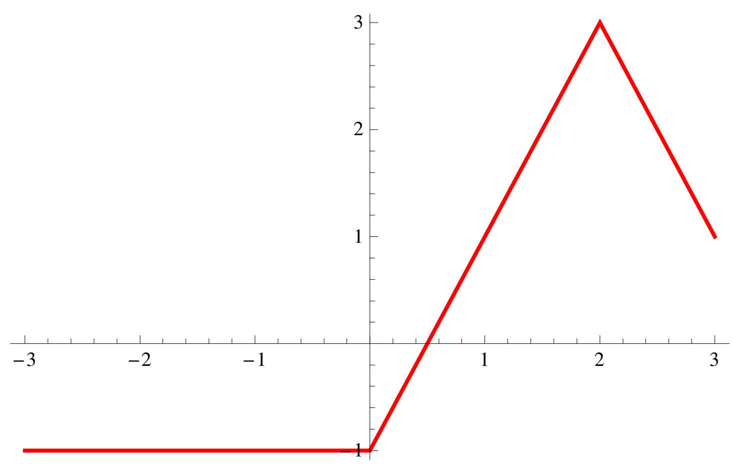
Sketching the Derivative Curve

In each of the problems below, the plot of some graph $y = f(x)$ is displayed. Use this plot to sketch the graph of the derivative $y = f'(x)$.

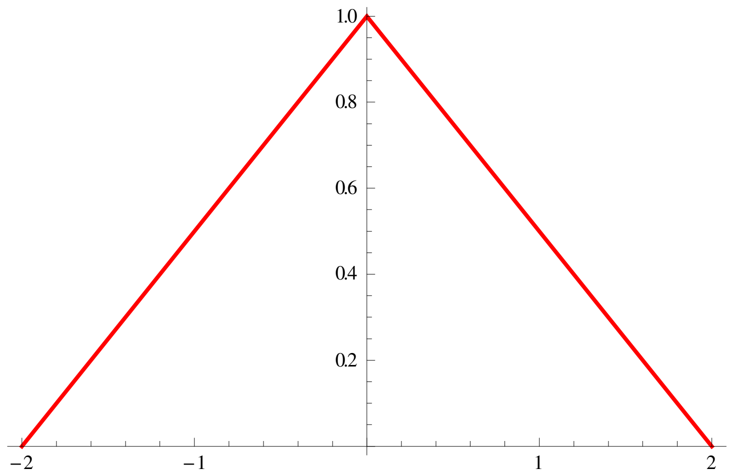
1.



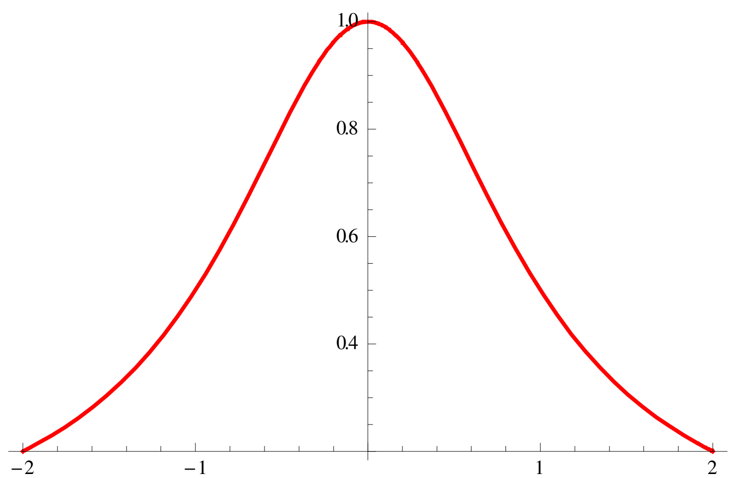
2.



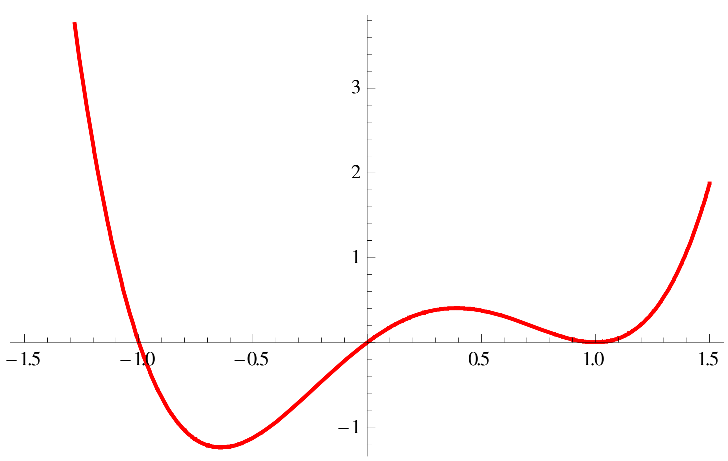
3.



4. Hint: When plotting the graphs of derivatives of curves, simply imagine that these curves are made up of many small line segments.



5.



Trigonometric Limits

Find the indicated limit if it exists. Do not use l'Hospital.

1. $\lim_{x\to 0} \frac{\sin 3x}{x}$
2. $\lim_{x\to 0} \frac{\sin 4x}{\sin 6x}$
3. $\lim_{t\to 0} \frac{\tan 6t}{\sin 2t}$
4. $\lim_{\theta\to 0} \frac{\tan a\theta}{\sin b\theta}$
5. $\lim_{x\to 0} \frac{\sin 2x}{x^2}$
6. $\lim_{h\to 0} \frac{\cos(h)-1}{\sin(h)}$
7. $\lim_{x\to 0} \frac{x\sin x}{1-\cos x}$
8. $\lim_{x\to 0} \frac{\sin 3x}{5x^3-4x}$
9. $\lim_{x\to 0} \frac{\sin(3x)\sin(5x)}{x^2}$
10. $\lim_{x\to 0} \frac{\sin(x^2)}{x}$
11. $\lim_{x\to 1} \frac{\sin(x-1)}{x^2+x-2}$
12. $\lim_{x\to \pi/4} \frac{1-\tan x}{\sin x-\cos x}$
13. $\lim_{x\to \infty} x\sin(1/x)$
14. $\lim_{x\to \infty} x\sin(5/x)$